Towards a theory of Warm Inflation of the Universe

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Abstract

The warm inflation scenario is an alternative mechanism which can explain the isotropic and homogeneous Universe which we are living in. In this work I extend a previously introduced formalism, without the restriction of slow - roll regime. Quantum to classical transition of the fluctuations is studied by means of the "transition function" here introduced. I found that the fluctuations of radiation energy density decrease with time and the thermal equilibrium at the end of inflation holds.

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I. INTRODUCTION

In the old inflation scenario [1], it was assumed that the Universe underwent isentropic expansion during the stage of rapid growth of the scale factor. However, this scenario predicts an inhomogeneous Universe due to the fact the temperature is larger than the critical one $T_c \sim 10^{15}$ GeV.

The chaotic inflation scenario describes a quasi - de Sitter expansion in a supercooled scenario. The entropy required to make the post-inflationary Universe consistent with observation is assumed to be generated in a short-time reheating period [2]. However, the fluctuations of temperature should be very large and many domains should surpass the critical temperature T_c. This fact would lead to a very inhomogeneous Universe.

Recently, Berera and Fang [3] showed how thermal fluctuations may play the dominant role in producing the initial perturbations during inflation. They invoked slow - roll conditions. This ingenious idea was extended in some papers [4] into the warm inflation scenario. This scenario served as an explicit demonstration that inflation can occur in presence of a thermal component. However, the radiation energy density ρ_r must be small with respect to the matter energy density ρ_φ. More exactly, the kinetic component of the energy density (ρ_{kin}) must be small with respect to the vacuum energy density. This condition is satisfied if

$$\rho_\varphi \sim V(\varphi) \gg \rho_r \gg \rho_{kin}, \tag{1}$$

where $V(\varphi)$ is the potential associated with the scalar field φ. A scenario of this kind provides a rapid expansion of the Universe in presence of a thermal component with small fluctuations of temperature compatible with the COBE data [5]. The thermal equilibrium is reached near the minimum of the potential $V(\varphi)$. Particles are created during this expansion and it is not necesary to have a further reheating era (like in standard inflation). More recently [6], J. Yokoyama and A. Linde showed that the solutions of warm inflation violate the adiabatic condition that the scalar field should not change significantly in the relaxation

time of particles interacting with it. They claim that if the energy released by the interaction of the field with the created particles is small, then the total number of particles in the warm universe must be very small and their interaction with the scalar field may be too small to keep it from rapid falling down.

According to the inflationary scenario, the inhomogeneities in the very early Universe are of genuine quantum origin [7]. But, at end of inflation the Universe becomes classical. The quantum to classical transition is due to accelerated expansion of the Universe and loss of coherence of the quantum fluctuations. This last is produced by the increment of the degrees of freedom of the coarse - grained field which characterizes the infrared sector, jointly with the dissipation produced by the interaction of the inflaton with the thermal bath [8–10].

In this paper, I extend the formalism of warm inflation recently introduced [9]. Exact solutions for the power - law expansion of the Universe are calculated for this formalism. The paper is organized as follow: in section II, I develope the formalism. In section III the potential inflation model is studied, and finally in section IV, the final comments and conclusions are presented.

II. THE FORMALISM

We consider a scalar field φ coupled minimally to a classical gravitational one with a Lagrangian:

$$\mathcal{L}(\varphi, \varphi_{,\mu}) = -\sqrt{-g}\left[\frac{R}{16\pi} + \frac{1}{2}g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} + V(\varphi)\right] + \mathcal{L}_{int}, \qquad (2)$$

where R is the scalar curvature, $g^{\mu\nu}$ the metric tensor, $V(\varphi)$ the scalar potential and g is the metric. The Lagrangian $\mathcal{L}_{int}$ takes into account the interaction of φ with other fields (i.e. bosons X,Y or fermions) of the thermal bath. The spacetime will be considered as isotropic and homogeneous, and characterized by a Friedmann - Robertson - Walker (FRW) metric

$$ds^2 = -dt^2 + a^2(t)\ dr^2. \qquad (3)$$

The scale factor $a(t)$, is a time dependent function which grows with time. The equation of motion for the operator φ is

$$\ddot{\varphi} - \frac{1}{a^2}\nabla^2\varphi + \left(3H(\varphi) + \tau(\varphi)\right)\dot{\varphi} + V'(\varphi) = 0, \tag{4}$$

where H is the Hubble parameter (the dot denotes the time derivative), $\tau(\varphi)\dot{\varphi}$ describes the density energy dissipated by the field φ into a thermalized bath, and the prime denotes the derivative with respect to φ.

The imposibility to solve the equation (4) becomes from our unknowledge of the Hilbert's space over which acts the field φ. For our proposal, will be sufficient to write the Friedmann equations in the semiclassical form

$$H^2 = \frac{8\pi}{3}G\left\langle \rho_\varphi + \rho_r \right\rangle, \tag{5}$$

where $G = M_p^{-2}$ is the gravitational constant, M_p is the Planckian mass ($M_p = 1.2\ 10^{19}$ GeV), and ρ_r and ρ_φ are the radiation and matter energy densities

$$\rho_\varphi = \frac{\dot{\varphi}^2}{2} + \frac{1}{a^2}\left(\vec{\nabla}\varphi\right)^2 + V(\varphi), \tag{6}$$

$$\rho_r = \frac{\tau}{8H}\dot{\varphi}^2. \tag{7}$$

In the conventional approach to the inflaton dynamics, the field φ is split into a spatially homogeneous classical piece plus a spatially inhomogeneous quantum piece that represents quantum fluctuations of the field[1]

$$\varphi(\vec{x},t) = \phi_c(t) + \phi(\vec{x},t). \tag{8}$$

We require that $< E|\varphi|E >= \phi_c(t)$ and $< E|\phi|E >= 0$, where $|E>$ is an arbitrary state.

A. Dynamics of the classical field ϕ_c

We define the classical field as a solution of the equation

[1] These fluctuations will be consider as very small.

$$\ddot{\phi}_c + (3H_c + \tau_c)\dot{\phi}_c + V'(\phi_c) = 0. \tag{9}$$

The Hubble parameter is expanded as:

$$H = H_c \left[1 + \frac{1}{2H_c^2} \left\langle \left(1 + \frac{\tau_c}{4H_c}\right) \dot{\phi}^2 + \frac{1}{a^2}(\vec{\nabla}\phi)^2 + \sum_n \frac{1}{n!} V^{(n+1)}(\phi_c)\ \phi^n \right\rangle \right], \tag{10}$$

where the classical Hubble parameter is

$$H_c^2 = \frac{4\pi}{3M_p^2} \left[\left(1 + \frac{\tau_c}{4H_c}\right) \dot{\phi}_c^2 + 2V(\phi_c) \right]. \tag{11}$$

The classical dynamics of ϕ_c and H_c are characterized by the following equations

$$\dot{\phi}_c = -\frac{M_p^2}{4\pi} H_c' \left(1 + \frac{\tau_c}{3H_c}\right)^{-1}, \tag{12}$$

$$\dot{H}_c = H_c'\ \dot{\phi}_c = -\frac{M_p^2}{4\pi} (H_c')^2 \left(1 + \frac{\tau_c}{3H_c}\right)^{-1}, \tag{13}$$

and the potential is

$$V(\phi_c) = \frac{3M_p^2}{8\pi} \left[H_c^2 - \frac{M_p^2}{12\pi} (H_c')^2 \left(1 + \frac{\tau_c}{4H_c}\right) \left(1 + \frac{\tau_c}{3H_c}\right)^{-2} \right], \tag{14}$$

where we have assumed $H(\varphi) = H(\phi_c) \equiv H_c$ and $\tau(\varphi) = \tau(\phi_c) \equiv \tau_c$. The expression for radiation energy density is

$$\rho_r \simeq \frac{\tau_c}{8H_c} \left(\frac{M_p^2}{4\pi}\right)^2 (H_c')^2 \left(1 + \frac{\tau_c}{3H_c}\right)^{-2}. \tag{15}$$

Near the minimum of the potential, the thermal equilibrium holds and the temperature is

$$< T_r > \sim \left(\frac{\tau_c(0)}{4H(\phi_c = 0)} \dot{\phi}_c^2 \right)^{1/4}. \tag{16}$$

In the inflationary epoch the kinetic energy is much smaller than the vacuum energy density $\rho_\varphi(\phi_c) \sim V(\phi_c)$. As in a previous work [9], I will consider the following relation between the Hubble parameter $H_c(\phi_c) = \dot{a}/a$ and the friction one $\tau_c(\phi_c)$:

$$\tau_c(\phi_c) = \gamma\ H_c(\phi_c). \tag{17}$$

This expression takes into account that the particlelike are dispersed during the expansion of the Universe. The rate of expansion of the Universe is given by the Hubble parameter $H_c(\phi_c)$

and the increment of the rate of expansion lead to an increment of the rate of friction. The dimensionless constant $\gamma = \tau_c/H_c$ in the equation (17) is a parameter of the theory which takes into account the intensity of friction due to the interaction between the inflaton and the fields of the thermal bath. Note that in a the de Sitter expansion $H_c(\phi_c)$ is constant (i.e., $H_c(\phi_c) = H_o$), and thus τ_c and ρ_r become zero ($\tau_c = \rho_r = 0$). In this case one recovers the standard inflation model [10]. The dynamics for the expansion and friction parameters is governed by the evolution of the classical field ϕ_c.

B. Dynamics of the Quantum Fluctuations

The equation of motion for the operator ϕ is

$$\ddot{\phi} - \frac{1}{a^2}\nabla^2\phi + (3H_c + \tau_c)\dot{\phi} + \sum_n \frac{1}{n!}V^{(n+1)}(\phi_c)\phi^n = 0, \tag{18}$$

where $V^{(n+1)}(\phi_c)$ denotes the $n+1$-th derivative. We assume that the quantum fluctuations are small and the Taylor expansion of $V'(\varphi)$ can be truncated at first order in ϕ

$$\ddot{\phi} - \frac{1}{a^2}\nabla^2\phi + (3H_c + \tau_c)\dot{\phi} + V''(\phi_c)\phi = 0. \tag{19}$$

For our propose, will be useful to consider the redefined field $\chi = e^{3/2\int (H_c+\tau_c/3)\ dt}\ \phi$. The equation of motion for this field is

$$\ddot{\chi} - a^{-2}\nabla^2\chi - \frac{k_o^2}{a^2}\chi = 0, \tag{20}$$

where

$$k_o^2 = a^2\left[\frac{9}{4}\left(H_c + \frac{\tau_c}{3}\right)^2 - V''(\phi_c) + \frac{3}{2}\left(\dot{H}_c + \frac{\dot{\tau}_c}{3}\right)\right], \tag{21}$$

is the squared time dependent wavenumber. The field χ can be written as a Fourier expansion of the modes $\xi_k(t)\ e^{i\vec{k}.\vec{r}}$

$$\chi = \frac{1}{(2\pi)^{3/2}}\int d^3k\left[a_k e^{i\vec{k}.\vec{r}}\xi_k(t) + h.c.\right], \tag{22}$$

where $\xi_k(t)$ are the time dependent modes. The annihilation and creation operators a_k and $a_k^\dagger$ satisfy the commutation relations

$$[a_k, a_{k'}^\dagger] = \delta(\vec{k} - \vec{k'}), \tag{23}$$

$$[a_k, a_{k'}] = [a_k^\dagger, a_{k'}^\dagger] = 0. \tag{24}$$

The equation of motion for ξ_k is

$$\ddot{\xi}_k(t) + \omega_k^2 \, \xi_k(t) = 0, \tag{25}$$

where $\omega_k^2 = a^{-2}(k^2 - k_o^2)$ is the squared frecuency of each mode with a given wavenumber k. The function k_o separates both, the unstable ($k^2 \ll k_o^2$) and the stable ($k^2 \gg k_o^2$) sectors. The quantum fluctuations with wavenumbers below k_o are interpreted as inhomogeneities superimposed on the classical field ϕ_c. These fluctuations are responsible for the density inhomogeneities generated during the inflation. The quantum field theory imposes the commutation relation between χ and $\dot{\chi}$

$$[\chi(\vec{r}, t), \dot{\chi}(\vec{r'}, t)] = \mathrm{i}\delta^{(3)} \, (\vec{r} - \vec{r'}). \tag{26}$$

The equation (26) implies that

$$\xi_k \dot{\xi}_k^* - \dot{\xi}_k \xi_k^* = \mathrm{i}. \tag{27}$$

To study the Universe on a scale greater than the scale of the observable Universe, we define the coarse - grained field that takes into account the long - wavelength modes. The wavelengths of such modes are

$$l \geq \frac{1}{\epsilon k_o}, \tag{28}$$

where $\epsilon \ll 1$ is a dimensionless constant. The coarse - grained field, so defined, is

$$\chi_{cg}(\vec{r}, t) = \frac{1}{(2\pi)^{3/2}} \int d^3k \; \theta(\epsilon k_o - k) \left[a_k e^{i\vec{k}.\vec{r}} \xi_k(t) + h.c. \right], \tag{29}$$

which satisfies the following operatorial stochastic equation

$$\ddot{\chi}_{cg} - \frac{k_o^2}{a^2}\chi_{cg} = \epsilon \left(\frac{d}{dt}(\dot{k}_o \eta) + 2\dot{k}_o \kappa \right), \tag{30}$$

with the operatorial noises

$$\eta(\vec{r},t) = \frac{1}{(2\pi)^{3/2}} \int d^3k \ \delta(\epsilon k_o - k) \quad \left[a_k e^{i\vec{k}.\vec{r}} \xi_k(t) + h.c \right], \tag{31}$$

$$\kappa(\vec{r},t) = \frac{1}{(2\pi)^{3/2}} \int d^3k \ \delta(\epsilon k_o - k) \quad \left[a_k e^{i\vec{k}.\vec{r}} \dot{\xi}_k(t) + h.c \right]. \tag{32}$$

The commutation relations are

$$[\chi_{cg}(t), \eta(t)] = 0, \tag{33}$$

$$[\chi_{cg}(t), \kappa(t)] = \frac{2\epsilon k_o}{(2\pi)^3} \int d^3k \quad \theta(\epsilon k_o - k)\delta(\epsilon k_o - k) \quad \left(\xi_k \dot{\xi}_k^* - \xi_k^* \dot{\xi}_k \right), \tag{34}$$

$$[\eta(t), \kappa(t)] = \frac{2\epsilon k_o}{(2\pi)^3} \int d^3k \quad \delta(\epsilon k_o - k)\delta(\epsilon k_o - k) \quad \left(\xi_k \dot{\xi}_k^* - \xi_k^* \dot{\xi}_k \right). \tag{35}$$

The correlations between the noises are

$$< \kappa(t)\kappa(t') >= \frac{1}{(2\pi)^3} \int d^3k \quad \delta(\epsilon k_o(t) - k) \quad \delta(\epsilon k_o(t') - k) \quad \left(\dot{\xi}_k(t) \dot{\xi}_k^*(t') \right), \tag{36}$$

$$< \eta(t)\eta(t') >= \frac{1}{(2\pi)^3} \int d^3k \quad \delta(\epsilon k_o(t) - k) \quad \delta(\epsilon k_o(t') - k) \quad \left(\xi_k(t) \xi_k^*(t') \right), \tag{37}$$

$$< \eta(t)\kappa(t') + \kappa(t)\eta(t') >= \frac{2}{(2\pi)^3} \int d^3k \quad \delta(\epsilon k_o(t) - k) \quad \delta(\epsilon k_o(t') - k) \quad \left(\dot{\xi}_k(t) \xi_k^*(t') \right). \tag{38}$$

Quantum to classical transition occurs due to the rapid expansion of the Universe during inflation. This transition is satisfied when the commutators (26), (33), (34) and (35) are nearly zero. We can write the time dependent modes as

$$\xi_k(t) = u_k(t) + \mathrm{i} \ v_k(t), \tag{39}$$

and the condition to obtain the complex to real transition of these modes is

$$\left| \frac{v_k(t)}{u_k(t)} \right| \ll 1. \tag{40}$$

We can define the quantum to classical transition function $\alpha_k(t) = \left| \frac{v_k(t)}{u_k(t)} \right|$. The modes are real when this function becomes nearly zero. The condition for the coarse - grained field $\chi_{cg}(\vec{r},t)$ to be classical is

$$\frac{1}{N(t)} \sum_{k=0}^{k=\epsilon k_o} \alpha_k(t) \ll 1, \tag{41}$$

where $N(t)$ is time dependent number of degrees of freedom of the infrared sector. During inflation, this number increases with time. This effect is due to the temporal evolution of both, the scale factor a and the superhorizon (with size $l \sim \frac{1}{\epsilon k_o}$). Quantum to classical transition of the coarse-grained field holds when $\alpha_{k=\epsilon k_o}(t) \rightarrow 0$.

Making $\tan[\Theta_k(t)] = \frac{v_k(t)}{u_k(t)}$, we can write the modes as

$$\xi_k(t) = e^{i\Theta_k(t)} \quad |\xi_k(t)|, \tag{42}$$

where $|\xi_k(t)| = \sqrt{v_k^2 + u_k^2}$. When the quantum fluctuations become classical, the correlations (36), (37) and (38) are

$$< \kappa(t)\kappa(t') >= \frac{1}{2\pi^2} \frac{\epsilon k_o^2}{\dot{k}_o} \xi_k(t)\xi_k(t') \quad \delta(t-t') \tag{43}$$

$$< \eta(t)\eta(t') >= \frac{1}{2\pi^2} \frac{\epsilon k_o^2}{\dot{k}_o} \dot{\xi}_k(t)\dot{\xi}_k(t') \quad \delta(t-t') \tag{44}$$

$$< \eta(t)\kappa(t') + \kappa(t)\eta(t') >= \frac{1}{2\pi^2} \frac{\epsilon k_o^2}{\dot{k}_o} \left(\xi_k(t)\dot{\xi}_k(t') + \dot{\xi}_k(t)\xi_k(t')\right) \quad \delta(t-t'). \tag{45}$$

C. Coarse-grained field correlations and power spectral density

Now we consider the correlation of χ_{cg} for different times t and t' ($t' > t$), once the modes are classical $[\xi_k(t)\xi_k^*(t') \simeq \xi_k(t)\xi_k(t')]$

$$< \chi_{cg}(t)\chi_{cg}(t') >= \frac{1}{(2\pi)^3} \int_{\epsilon k_o(t)}^{\epsilon k_o(t')} d^3k \quad \xi_k(t)\xi_k(t'). \tag{46}$$

The Fourier transform of $< \chi_{cg}(t)\chi_{cg}(t') >$ gives the power spectral density for χ_{cg} in the infrared sector

$$S[\chi_{cg};\omega_k] = 4 \int_0^\infty cos[\omega t''] \quad |< \chi_{cg}(t)\chi_{cg}(t+t'') >|_{t\gg 1} \quad dt'', \tag{47}$$

where $t'' = t' - t$. This expression, written explicitly, becomes

$$S[\chi_{cg};\omega_k] = \frac{1}{\pi} \int_0^\infty dt'' \quad cos[\omega t''] \quad \int_{\epsilon k_o(t)}^{\epsilon k_o(t+t'')} dk \ k^2 \ \xi_k(t)\xi_k(t+t''), \tag{48}$$

where ω_k is the oscillation frecuency for a given wavenumber k. The equation (48) is only valid when the thermal equilibrium holds. The fluctuations of radiation energy density are

$$\frac{\delta\rho_r}{\rho_r} \simeq \left|2(H_c')^{-1}\ H_c''\right| \quad < \phi_{cg}^2 >^{1/2}, \tag{49}$$

where $\phi_{cg}(\vec{r},t) = a^{-1/2(3+\gamma)}\ \chi_{cg}(\vec{r},t)$. The condition for the thermal equilibrium holds is $\frac{d}{dt}\left(\frac{\delta\rho_r}{\rho_r}\right) < 0$.

III. POWER - LAW INFLATION

This model of inflation is characterized by $a(t)$ and H_c given by

$$a(t) = H_o^{-1}\ \left(\frac{t}{t_o}\right)^p, \tag{50}$$

$$H_c(t) = \frac{p}{t}. \tag{51}$$

The temporal evolution of the classical field is

$$\phi_c(t) = \phi_o - m\ln\ \left[\frac{H_o}{p}t\right]. \tag{52}$$

The solution of the equation (15) for the radiation energy density is

$$\rho_r = \frac{\gamma M_p^4\ p^2}{128\pi^2 m^2(1+\gamma/3)^2}\ t^{-2}. \tag{53}$$

The radiation temperature is

$$< T_r > \propto M_p m^{-1/2}\ t^{-1/2}, \tag{54}$$

where m is the mass of the scalar field. The potential for this model is

$$V(\phi_c) = \frac{3M_p^2}{8\pi} H_c^2 \left\{1 - \frac{M_p^2}{48\pi} m^{-2}\left[(1+\gamma/4)\,(1+\gamma/3)^{-2}\right]\right\}. \tag{55}$$

The equation of motion for the modes is

$$\ddot{\xi}_k(t) + \left[\frac{H_o^2 t_o^{2p} k^2}{t^{2p}} - \mu^2(t)\right] \quad \xi_k(t) = 0, \tag{56}$$

with

$$\mu^2(t) = t^{-2}K^2, \tag{57}$$

and

$$K^2 = \left[\frac{9}{4}p^2(1+\gamma/3)^2 - \frac{3}{2}p(1+\gamma/3) + \frac{3p^2M_p^2}{2\pi m^2}\left(1 - \frac{M_p^2}{48\pi^2 m^2}(1+\gamma/4)(1+\gamma/3)^{-2}\right)\right]. \tag{58}$$

For inflation takes place, we must take $k_o^2 > 0$, and the expansion must be sufficiently rapid such that $p > \frac{3}{2A}(1+\gamma/3)$, with $A = 9/4(1+\gamma/3)^2 + \frac{3M_p^2}{2\pi m^2}\left(1 - \frac{M_p^2}{48\pi m^2}(1+\gamma/4)(1+\gamma/3)^{-2}\right)$. The general solution for the equation (56) is

$$\xi_k(t) = A_1\sqrt{t}\ H_\nu^{(1)}(t)\left[\frac{H_o k(t/t_o)^{1-p}}{p-1}\right] + A_2\sqrt{t}\ H_\nu^{(2)}(t)\left[\frac{H_o k(t/t_o)^{1-p}}{p-1}\right], \tag{59}$$

with $\nu = \frac{1}{2(p-1)}\sqrt{1+4K^2}$. When $\gamma = 0$ (i.e., for $\tau_c = 0$), we recover the results of standard inflation [10]. We choose the Bunch - Davis vacuum ($A_1 = 0$), but with imaginary constant $A_2 = \mathrm{i}|A_2|$ that can be written in terms of the Bessel functions

$$\xi_k(t) = \frac{\sqrt{\pi}}{2}\frac{\sqrt{t/t_o}}{p-1}\left[\mathcal{Y}_\nu[x(t)] + \mathrm{i}\mathcal{J}_\nu[x(t)]\right] = \mathrm{i}\ H_\nu^{(2)}[x(t)], \tag{60}$$

where $H_\nu^{(2)}[x(t)]$ is the Hankel function and $x(t) = \frac{H_o k(t/t_o)^{1-p}}{p-1}$. Observe that $x(t)$ tends to zero when t is sufficiently large. In this case one obtains $x(t) \ll 1$, and the asymptotic modes are real

$$\xi_k(t) \simeq \frac{\sqrt{\pi}}{2}\frac{\sqrt{t/t_o}}{p-1}\mathcal{Y}_\nu\left[x(t)\right], \tag{61}$$

which is due to

$$\left|\frac{\mathcal{J}_\nu[x(t)]}{\mathcal{Y}_\nu[x(t)]}\right| \ll 1. \tag{62}$$

So, for $(t/t_o)^{p-1} \gg \frac{H_o k}{p-1}$ the modes with the wavenumber k are real. For $x(t) \ll 1$ the function $\mathcal{Y}_\nu$ becomes

$$\mathcal{Y}_\nu \simeq -\frac{\mathrm{i}}{\pi}\Gamma(\nu)\left(\frac{H_o k(t/t_o)^{1-p}}{2(p-1)}\right)^{-\nu}, \tag{63}$$

where $\Gamma(\nu)$ is the gamma function with argument ν. The correlation of the coarse - grained field, for $t \gg 1$, is

$$< \phi_{cg}(t=t_o)\phi_{cg}(t_1) >|_{p\gg 1} \simeq \frac{2\pi 4^{\nu}\Gamma^2(\nu)}{(p-1)^4}\left[\epsilon K\right]^{2(1-\nu)+1} \left(\frac{t_1}{t_o}\right)^{4\nu p(1-\nu/p-p/\nu)+3-\gamma}, \qquad (64)$$

where K is given by the equation (58).

The power spectral density of the fluctuations (when the thermal equilibrium holds) is the Fourier transform of the equation (64), which becomes [see eqs. (48) and (61)]

$$S[\chi_{cg};\omega_k = \frac{kH_o}{(t_1/t_o)^p}] \simeq \frac{8\pi 4^{\nu}\Gamma^2(\nu)}{(p-1)^4}\left[\epsilon K\right]^{2(1-\nu)+1}\cos\left[\frac{1}{2}\pi\left(4\nu(p-1)-2p+4\right)\right] \\ \times \Gamma\left[\left(4\nu(p-1)-2p+4\right)\right]|\omega_k|^{-(4\nu(p-1)-2p+4)}. \qquad (65)$$

For $p \gg 1$ one obtains $S[\chi_{cg},\omega_k] \propto |\omega_k|^n$, with $n = -(4\nu(p-1)-2p+4)$. The fluctuations of the radiation energy density are

$$\left.\frac{\delta\rho_r}{\rho_r}\right|_{t\gg 1} \sim t^{2\nu p(1-\nu/p-\frac{5}{4}p/\nu)+\frac{1}{2}-\frac{\gamma}{2}}. \qquad (66)$$

Note that both, $< \phi_{cg}(t_o)\phi_{cg}(t_1) >$ and $\frac{\delta\rho_r}{\rho_r}$ decreases with time for sufficiently large p.

IV. CONCLUSIONS

In this work, I extended the formalism for warm inflation introduced in a previous paper. In the model the rapid expansion of the Universe is produced with a temperature T_r smaller than T_c. The dynamics of the fluctuations of the matter field generates thermal fluctuations which decrease with time. So, at the end of inflation, the thermal equilibrium holds. On the other hand, the classical field $\phi_c(t)$ is responsible for the expansion of the Universe and the dissipation of energy. The dissipation of energy is due to the interaction between the inflaton and the particles in the thermal bath (with mean temperature $< T_r > < T_c$). The dynamics of the interaction is represented by the classical parameter $\tau_c(\phi_c)$. This parameter is considered as proportional to the rate of expansion of the Universe, which is given by the Hubble parameter $H_c(\phi_c)$.

This model has four foundamental aspects: **1)** The classical one: characterized by the size of the superhorizon $l(t) \sim \frac{1}{\epsilon k_o}$. The quantum to classical transition of the fluctuations in the

infrared sector is due to the complex to real transition of the modes ξ_k of χ_{cg}. The dynamics of this transition is described by the sum over k in the infrared sector, for the function $\alpha_k(t)$. **2)** The stochastic ingredient of the coarse - grained field: the fluctuations of the matter field χ_{cg} describes the infrared sector and and its evolution is governed by a classical stochastic equation. **3)** The thermal ingredient: the dissipation parameter τ_c represents the interaction of the inflaton with a thermal bath. In this model, when the inflation starts the Universe is with a mean temperature smaller than the GUT's temperature ($T_c \sim 10^{15}$ GeV). In the example for power - law inflation here studied, the mean temperature decreases as $t^{-1/2}$ and the thermal fluctuations also decrease for sufficiently large p. So, at the end of inflation the Universe is more cold than when inflation starts. The thermal equilibrium holds due to the decreasing with time of the fluctuations for the radiation energy density. Due to this fact, it is possible to calculate the power spectral density for χ_{cg}. This spectrum depends on the rate of expansion of the Universe and the interaction of the inflaton with the thermal bath. **4)** The quantum aspect: relies in the time dependent modes $\xi_k(t)$ of the coarse - grained field χ_{cg}. These modes must be real in the infrared sector to give classical fluctuations.

Initially (i.e., when the inflation starts), the fluctuations $\phi(\vec{x},t)$ [and also $\chi(\vec{x},t)$] are quantized, since the energy density of the false vacuum is of the order (but less) of the Planckian scale ($\rho_\varphi \sim V(\varphi) < M_p^4$). Furthermore the amplification of the modes that enters in the infrared sector generates loss of coherence of the field χ_{cg}. More rapid is the expansion of the Universe, more rapid is the complex to real transition of each mode with a wavenumber k. The modes ξ_k with smaller wavenumber will become real before the bigger ones. So, the Universe becomes classical in the infrared sector.

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